AD-A095 058 VIRGINIA POLYTECHNIC INST AND STATE UNIV BLACKSBURG F/G 12/1

SEQUENTIAL TESTS UNDER WEIBULL DISTRIBUTION.(U)

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SEQUENTIAL TESTS UNDER WEIBULL DISTRIBUTION

RESEARCH REPORT NO. Q-7

P. M. GHARE

DECEMBER 1980

U. S. ARMY RESEARCH OFFICE

GRANT NO. DAAG-29-78-G-0172

VIRGINIA POLYTECHNIC INSTITUTE
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SEQUENTIAL TESTS UNDER WEIBULL DISTRIBUTION

by

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Reliability and Maintainability Symposium

Philadelphia January 1981

ABSTRACT

The objective of this paper is to develop acceptance procedures based on a sequential probability ratio test when the system failures follow Weibull probability distribution. The acceptance procedures in both Military Handbook 108 and Mil-Std-781C are based on exponential distribution of failures. Exponential distribution implies a constant hazard rate. The Weibull distribution is much more comprehensive and, with suitable choice of parameters, can describe constant, increasing or decreasing hazard rates.

The specifications are usually written in terms of the scale parameter α, the shape parameter β being determined by the nature of the system under test. When N units are being tested and n units have failed at time t, the acceptance is based on the monitoring of a statistic z_t given as

$$z_t = -n(\log \alpha_1 - \log \alpha_0) + \frac{\alpha_1-\alpha_0}{\beta+1}\left[\sum_{i=1}^{n} t_i^{\beta+1} + (N-n)t^{\beta+1}\right]$$

where α_0 and α_1 are the desirable and unacceptable values of the scale parameters. The indicated decision is to

accept if $z_t \geq$ a constant, $\log C_1$

reject if $z_t \leq$ another constant, $\log C_2$

and to continue testing if z_t remains within the two limits.

SEQUENTIAL TESTS UNDER WEIBULL DISTRIBUTION

I. Objective

The objective of this paper is the development of acceptance procedures based on a sequential probability ratio test when the system failures follow a Weibull probability distribution. The acceptance procedures in both Military Handbook H 108 and standard Mil-Std-781C are based on an exponential distributions of failures.

There are two major reasons for attempting to base acceptance procedures on a Weibull distribution rather than on an exponential distribution.

1. Weibull distribution is more comprehensive than exponential. Exponential distribution would only apply to those systems where the hazard rate remains constant over time. Weibull distribution would, with suitable choice of parameters, be applicable to all systems where the hazard rate is constant, increasing, or decreasing.
2. The model describing the growth of the reliability of a new system under development can be shown to be a special case of the Weibull model. The growth model, first studied by Duane (1), describes a sequence of changing system configurations and is different from the static reliability models for a fixed configuration. However the model itself follows a probability distribution which can be approximated by Weibull distribution with shape parameter $\theta = -1/2$.

The principal reason for a sequential probability ratio test is that such test procedures involve a minimum amount of testing.

II. Probability Ratio Tests for Multi-parameter Functions

A. Symbols

Let $f(x, \theta_1, \ldots, \theta_t, \phi_1, \ldots, \phi_k)$ be probability density function where

x = a discrete valued variable

θ_i, ϕ_j = parameters

T = a t-component vector $\theta_1, \ldots, \theta_t$

K = a k-component vector $\phi_1, \ldots, \phi_k$

Ω_T = parameter space for parameters $\theta_1, \ldots, \theta_k$

Ω_K = parameter space for parameters $\phi_1, \ldots, \phi_k$

a,b = subspaces of Ω_T and Ω_K

$$L = \prod_{i=1}^{n} f(x_i, T, K)$$

$L^*(a,b)$ = max $L | T\varepsilon a,\ K\varepsilon b$

$\lambda(a,b)$ = probability ratio

$$= \frac{L^*(a,b)}{L^*(\Omega_T, \Omega_K)}$$

B. General Probability Ratio Test

The parameter set T is the set under study. Set K includes all other parameters. Hence all hypotheses are stated in terms of T. There will be 3 cases of the tests of hypotheses according to the assumption about K.

Under assumptions of $f(\cdot)$ satisfying general regularity conditions and large n, in a test of the hypothesis

$$H_0: \quad T = T^0$$

the quantity $-2 \log \lambda(T^0, \Omega_K)$ is approximately distributed as chi-square with t degrees of freedom when H_0 is true.

This approximate distribution can be used to devise a test of the hypothesis

$$H_0: \quad T = T^0$$

against a composite hypothesis

$$H_1: \quad T \neq T^0$$

The power of this test is not easy to determine as the distribution of $-2 \log \lambda(T^1, \omega_K)$ is non-central chi-square.

C. Special Cases of Probability Ratio Tests

Three special cases of probability ratio tests can be defined according to the assumptions regarding K.

Case 1 K is assumed to be known.

Case 2 An estimate $\hat{K}$ is used in place of K.

Case 3 K is unspecified.

Case 1: This case involves the testing of a simple hypothesis

$$H_0: \quad T = T^0$$

against a simple hypothesis

$$H_1: \quad T = T^1$$

with an added condition that K is known to be equal to a constant $\bar{K}$. In this case the parameter space under both H_0 and H_1 would be single points and

$$L^*(\cdot,\cdot) = L$$

For this case a test, with specified values α and β of Type I and Type II errors, can be devised following Wald (4).

If H_0 is true and H_1 is false, then

$$L(T^0, \bar{K}) \geq 1-\alpha$$

and

$$L(T^1,\bar{K}) \leq \beta$$

Therefore the test statistic s_1 satisfies

$$s_1 = \frac{L(T^0,\bar{K})}{L(T^1,\bar{K})} \geq \frac{1-\alpha}{\beta} \qquad 2.1$$

If H_0 is false and H_1 is true, then

$$L(T^0,\bar{K}) \leq \alpha$$

and

$$L(T^1,\bar{K}) \geq 1-\beta$$

Therefore the test statistic s_1 satisfies

$$s_1 \leq \frac{\alpha}{1-\beta} \qquad 2.2$$

Inequalities (2.1) and (2.2) define the acceptance and rejection regions respectively. When s_1 lies between $\frac{\alpha}{1-\beta}$ and $\frac{1-\alpha}{\beta}$ there is no decision. In this case, the sample size can be sequentially increased until an accept or reject decision is reached. Such a test would be called a Sequential Probability Ratio Test, SPRT.

Sequential Probability Ratio Test was first developed by A. Wald (4). The optimality of SPRT was formally proved by A. Wald and J. Wolfowicz (5). The problem of optimality of SPRT for Markov processes was studied by Ghosh

(2). A. Sirjaev (3) considers the more general optimal stopping problem for Markov processes. Sirjaev (3) also gives a discrete time formulation of SPRT.

Case 2: This case also would be like Case 1 but with an assumed value $\hat{K}$ being used instead of the true value $\overline{K}$. In this case the computed values of L would be in error and these errors can be defined as error factors.

$$e_0 = \frac{L(T^0,\hat{K})}{L(T^0,\overline{K})}$$

and

$$e_1 = \frac{L(T^1,\hat{K})}{L(T^1,\overline{K})}$$

Then the test statistic s_3 would be given by

$$s_3 = \frac{e_0}{e_1} \cdot s_2$$

and it would be possible to define the acceptance and rejection regions of the type

accept if $s_3 \geq q_u$

reject if $s_3 \leq q_L$

and increase sample size if $q_L < s_3 < q_u$.

An approximating distribution of the statistic s_3 would have to be developed to obtain the Type I and Type II errors for this test.

Case 3: This case involves the testing of a simple hypothesis

$$H_0: \quad T = T^0$$

against a simple hypothesis

$$H_1: \quad T = T^1$$

Consider the test statistic s_2 given by

$$s_2 = \frac{\lambda(T^0,\Omega_K)}{\lambda(T^1,\Omega_K)} = \frac{L^*(T^0,\Omega_K)}{L^*(\Omega_T,\Omega_K)} \cdot \frac{L^*(\Omega_T,\Omega_K)}{L^*(T^1,\Omega_K)}$$

If H_0 is true then $L^*(T^0,\Omega_K) \geq 1-\alpha$ and

$$L^*(T^1,\Omega_K) \leq \beta.$$

Consequently

$$s_2 \geq \frac{1-\alpha}{\beta}$$

Similarly when H_1 is true

$$s_2 \leq \frac{\alpha}{1-\beta}$$

Again, as in the cases (1) and (2) these two inequalities define the acceptance and rejection regions. When s_2 lies between $\frac{\alpha}{1-\beta}$ and $\frac{1-\alpha}{\beta}$ there is no decision and the sample size can be sequentially increased until an accept or reject decision is reached.

III. Reliability Tests Using Weibull Distribution

A. Weibull distribution is a 3 parameter distribution with parameters

q = scale parameter

θ = shape parameter

and γ = location parameter

For reliability studies the location parameter is known. In "time-to-failure" tests γ is always zero. In the "stress-to-failure" tests γ is known from engineering considerations. Then the Weibull distribution can be transformed to a two parameter distribution by counting the time (stress) as $t-\gamma$ instead of t.

Further the reliability specifications are written in terms of MTTF. For a specified value of θ these can be seen as specifications for q. The Table 6-1 in the appendix gives the values of α corresponding to the values of MTTF for specified θ.

A good estimate of θ is almost always available from historic data about similar projects. In the proposed acceptance procedure the historic estimate of θ can be used in place of the known θ and this assumption can be validated by testing an associate hypothesis H_0: $\theta = \hat{\theta}$ at frequent intervals during the testing.

Although it is assumed here that a historic estimate of θ is available, there can be circumstances where such estimates of θ are not available. For example a totally new system may not have suitable data available. In these cases the analysis would follow "case 3" in the preceding section. It would be difficult to obtain any operational procedures, however, as the computation of $L^*(T^0, \Omega_K)$ would not be as straightforward as the computation of $L(T^0, \bar{K})$. This case is not addressed to in the following operational procedure.

B. Symbols

q_0: An acceptable value of q

q_1: An unacceptable value of q

θ: Historic shape parameter estimate

t_i: Time to the i^{th} failure

t: Time

N: Number of units being tested

n: Number of units failed

C. Test of the Hypothesis

The test of the hypothesis

$$H_0: \; q = q_0$$

against

$$H_1: \; q = q_1$$

under the assumption of θ being known would come under Case 1. Here $T = \{q\}$ and $K = \{\theta\}$. For the stipulated Type I and Type II error probabilities of α and β respectively, the test itself will take the form

$$\text{accept if } s_1 \geq \frac{1-\alpha}{\beta}$$

$$\text{reject if } s_1 \leq \frac{\alpha}{1-\beta}$$

and continue testing if $\frac{\alpha}{1-\beta} < s_1 < \frac{1-\alpha}{\beta}$

$$L(T^0,\bar{K}) = [\prod_{i=1}^{n} q_0 t_i^{\theta} \operatorname{Exp}(-q_0 t_i^{\theta+1}/\theta+1)] \cdot \operatorname{Exp}(-(N-n)q_0 t^{\theta+1}/\theta+1)$$

and

$$L(T^1,\overline{K}) = [\prod_{i=1}^{n} q_1 t_i^{\theta} \mathrm{Exp}(-q_1 t_i^{\theta+1}/\theta+1)] \cdot \mathrm{Exp}((N-n) q_1 t^{\theta+1}/\theta+1)$$

Taking logarithms and simplifying

$$\log s_1 = \log \frac{L(T^0,\overline{K})}{L(T^1,\overline{K})}$$

$$= n(\log \alpha_0 - \log \alpha_1) + \frac{q_1 - q_0}{\theta+1} [\sum_{i=1}^{n} t_i^{\theta+1} + (N-n)t^{\theta+1}]$$

Let $z_1 = \frac{1-\alpha}{\beta}$ and $z_2 = \frac{\alpha}{1-\beta}$. Then the acceptance condition becomes

$$\log s_1 \geq \log z_1$$

and the rejection condition becomes

$$\log s_1 \leq \log z_2$$

Further algebraic simplification leads to the conditions based on another statistic

$$s^*(t) = \sum_{i=1}^{n} t_i^{\theta+1} + (N-n)t^{\theta+1}$$

so that

$$\log s_1 = -n(\log q_1 - \log q_0) + \frac{q_1 - q_0}{\theta + 1} \; s^*(t)$$

$$= \sigma$$

leading to

accept if $\sigma \geq \log z_1$

reject if $\sigma \leq \log z_2$

and continue sampling if σ remains within the two limits.

D. Computational Procedure

It can be noticed that the right hand sides of the two conditions are constant. These can be plotted as two horizontal lines and σ can be plotted as a function of time.

At any interval when there is no failure σ would be decreasing and there would be sharp upward jumps whenever there is a failure. The test chart would appear as given in Figure (1).

It can be further observed that when there is no failure, the term $-n(\log q_1 - \log q_0)$ remains constant and the term $\frac{q_1 - q_0}{\theta + 1} s^*(t)$ in an increasing function of time. The slope of this function at time t can be calculated as

$$\frac{d}{dt}\left[\frac{q_1 - q_0}{\theta + 1} s^*(t)\right] = \frac{q_1 - q_0}{\theta + 1} \frac{d}{dt} s^*(t)$$

and

$$\frac{d}{dt} s^*(t) = \frac{d}{dt}\left[\sum_1^n t_i^{\theta+1} + (N-n)t^{\theta+1}\right]$$

$$= (N-n)(\theta+1)t^{\theta}$$

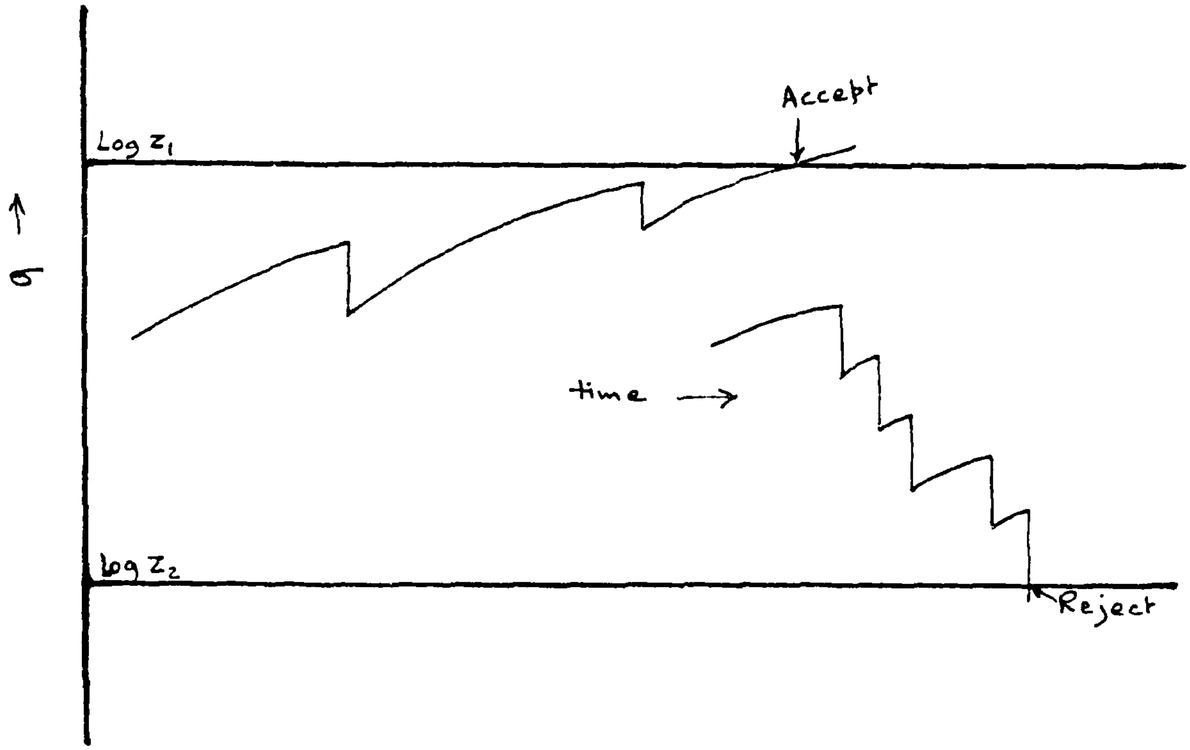

Figure 1

Sequential Acceptance Chart

$$\text{slope} = \frac{q_1 - q_0}{\theta + 1}\,(N-n)(\theta+1)t^{\theta}$$

$$= (q_1 - q_0)(N-n)t^{\theta}$$

At every failure epoc the first term would decrease by the quantity $(\log q_1 - \log q_0)$.

To facilitate the computations the quantities t^{θ} have been tabulated for selected values of θ in Table (6-2) in the appendix.

IV. Operational Procedure

The operational procedure for sequential tests under Weibull distribution can be summarized as follows.

1) Determine the desired and unacceptable values of MTTF.

2) Obtain an estimate of θ. Preferably choose an acceptable θ value out of the following

   -1, -0.5, 0, 0.5, 1, 2, 3.

   Almost any practical failure density can be closely approximated by one of these values.

3) Obtain q_1 and q_0 from Table (6.1).

4) Determine N the number of units to be tested.

5) Maintain and update the chart, similar to Figure (1).

Note: When a large number of units are to be tested it would be desirable to revalidate the estimate of θ at suitable intervals.

VI. Appendix

Table 6.1 Values of q for stated MTTF

Table 6.2 Values of t^{θ}

References

1. Duane, J. T., "Learning Curve Approach to Reliability Monitoring", IEEE Transactions on Aerospace, Vol. 2, April 1964.

2. Ghosh, B. K., Sequential Tests of Statistical Hypotheses, Addison-Wesley, Reading, Massachusetts, 1970.

3. Sirjaev, A. N., Statistical Sequential Analysis, American Mathematical Society, Providence, 1973.

4. Wald, A., Sequential Analysis, John Wiley, New York, 1947.

5. Wald, A. and Wolfowitz, J., "Optimum Character of Sequential Probability Ratio Test", Annals of Mathematical Statistics, Vol. 19, 1948.

Table 6.1

Values of Parameter q for Selected MTTF Value

m	θ = -0.50		θ = 0.00		θ = 1.00		θ = 2.00		θ = 3.00	
1	0.707		0.100	1	0.157	1	0.213	1	0.269	1
2	0.500		0.500		0.392		0.267		0.168	
3	0.408		0.333		0.174		0.791	-1	0.333	-1
4	0.353		0.250		0.981	-1	0.333	-1	0.105	-1
5	0.316		0.200		0.628	-1	0.170	-1	0.431	-2
6	0.288		0.166		0.436	-1	0.988	-2	0.208	-2
7	0.267		0.142		0.320	-1	0.622	-2	0.112	-2
8	0.250		0.125		0.245	-1	0.417	-2	0.659	-3
9	0.235		0.111		0.193	-1	0.293	-2	0.411	-3
10	0.223		0.100		0.157	-1	0.213	-2	0.269	-3
15	0.182		0.666	-1	0.698	-2	0.632	-3	0.533	-4
20	0.158		0.500	-1	0.392	-2	0.267	-3	0.168	-4
25	0.141		0.400	-1	0.251	-2	0.136	-3	0.691	-5
30	0.129		0.333	-1	0.174	-2	0.791	-4	0.333	-5
35	0.119		0.285	-1	0.128	-2	0.498	-4	0.179	-5
40	0.111		0.250	-1	0.981	-3	0.333	-4	0.105	-5
45	0.105		0.222	-1	0.775	-3	0.234	-4	0.658	-6
50	0.100		0.200	-1	0.628	-3	0.170	-4	0.431	-6
55	0.953	-1	0.181	-1	0.519	-3	0.128	-4	0.295	-6
60	0.912	-1	0.166	-1	0.436	-3	0.988	-5	0.208	-6
65	0.877	-1	0.153	-1	0.371	-3	0.777	-5	0.151	-6
70	0.845	-1	0.142	-1	0.320	-3	0.622	-5	0.112	-6
75	0.816	-1	0.133	-1	0.279	-3	0.506	-5	0.853	-7
80	0.790	-1	0.125	-1	0.245	-3	0.417	-5	0.659	-7
85	0.766	-1	0.117	-1	0.217	-3	0.347	-5	0.517	-7
90	0.745	-1	0.111	-1	0.193	-3	0.293	-5	0.411	-7
95	0.725	-1	0.105	-1	0.174	-3	0.249	-5	0.331	-7
100	0.707	-1	0.100	-1	0.157	-3	0.213	-5	0.269	-7
200	0.500	-1	0.500	-2	0.392	-4	0.267	-6	0.168	-8
300	0.408	-1	0.333	-2	0.174	-4	0.791	-7	0.333	-9
400	0.353	-1	0.250	-2	0.981	-5	0.333	-7	0.105	-9
500	0.316	-1	0.200	-2	0.628	-5	0.170	-7	0.431	-10
600	0.288	-1	0.166	-2	0.436	-5	0.988	-8	0.208	-10
700	0.267	-1	0.142	-2	0.320	-5	0.622	-8	0.112	-10
800	0.250	-1	0.125	-2	0.245	-5	0.417	-8	0.659	-11
900	0.235	-1	0.111	-2	0.193	-5	0.293	-8	0.411	-11
1000	0.223	-1	0.100	-2	0.157	-5	0.213	-8	0.269	-11
2000	0.158	-1	0.500	-3	0.392	-6	0.267	-9	0.168	-12
3000	0.129	-1	0.333	-3	0.174	-6	0.791	-10	0.333	-13
4000	0.111	-1	0.250	-3	0.981	-7	0.333	-10	0.105	-13
5000	0.100	-1	0.200	-3	0.628	-7	0.170	-10	0.431	-14
6000	0.912	-2	0.166	-3	0.436	-7	0.988	-11	0.208	-14
7000	0.845	-2	0.142	-3	0.320	-7	0.622	-11	0.112	-14
8000	0.790	-2	0.125	-3	0.245	-7	0.417	-11	0.659	-15
9000	0.745	-2	0.111	-3	0.193	-7	0.293	-11	0.411	-15
10000	0.707	-2	0.100	-3	0.157	-7	0.213	-11	0.269	-15

Note: In this table the values of q are listed in the form $x*10^{y}$. For example |0.125|2| means $0.125*10^{2}$ or 12.5.

Table 6.2

Values of t^{ϑ}

t	ϑ = -0.50		ϑ = 0.00		ϑ = 1.00		ϑ = 2.00		ϑ = 3.00	
1	0.100	1	0.100	1	0.100	1	0.100	1	0.100	1
2	0.707		0.100	1	0.200	1	0.400	1	0.800	1
3	0.577		0.100	1	0.300	1	0.900	1	0.270	2
4	0.500		0.100	1	0.400	1	0.160	2	0.640	2
5	0.447		0.100	1	0.500	1	0.250	2	0.125	3
6	0.408		0.100	1	0.600	1	0.360	2	0.216	3
7	0.377		0.100	1	0.700	1	0.490	2	0.343	3
8	0.353		0.100	1	0.800	1	0.640	2	0.512	3
9	0.333		0.100	1	0.900	1	0.810	2	0.729	3
10	0.316		0.100	1	0.100	2	0.100	3	0.100	4
15	0.258		0.100	1	0.150	2	0.225	3	0.337	4
20	0.223		0.100	1	0.200	2	0.400	3	0.800	4
25	0.200		0.100	1	0.250	2	0.625	3	0.156	5
30	0.182		0.100	1	0.300	2	0.900	3	0.270	5
35	0.169		0.100	1	0.350	2	0.122	4	0.428	5
40	0.158		0.100	1	0.400	2	0.160	4	0.640	5
45	0.149		0.100	1	0.450	2	0.202	4	0.911	5
50	0.141		0.100	1	0.500	2	0.250	4	0.125	6
55	0.134		0.100	1	0.550	2	0.302	4	0.166	6
60	0.129		0.100	1	0.600	2	0.360	4	0.216	6
65	0.124		0.100	1	0.650	2	0.422	4	0.274	6
70	0.119		0.100	1	0.700	2	0.490	4	0.343	6
75	0.115		0.100	1	0.750	2	0.562	4	0.421	6
80	0.111		0.100	1	0.800	2	0.640	4	0.512	6
85	0.108		0.100	1	0.850	2	0.722	4	0.614	6
90	0.105		0.100	1	0.900	2	0.810	4	0.729	6
95	0.102		0.100	1	0.950	2	0.902	4	0.857	6
100	0.100		0.100	1	0.100	3	0.100	5	0.100	7
200	0.707	-1	0.100	1	0.200	3	0.400	5	0.800	7
300	0.577	-1	0.100	1	0.300	3	0.900	5	0.270	8
400	0.500	-1	0.100	1	0.400	3	0.160	6	0.640	8
500	0.447	-1	0.100	1	0.500	3	0.250	6	0.125	9
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4000	0.158	-1	0.100	1	0.400	4	0.160	8	0.640	11
5000	0.141	-1	0.100	1	0.500	4	0.250	8	0.125	12
6000	0.129	-1	0.100	1	0.600	4	0.360	8	0.216	12
7000	0.119	-1	0.100	1	0.700	4	0.490	8	0.343	12
8000	0.111	-1	0.100	1	0.800	4	0.640	8	0.512	12
9000	0.105	-1	0.100	1	0.900	4	0.810	8	0.729	12
10000	0.100	-1	0.100	1	0.100	5	0.100	9	0.100	13

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4. TITLE (and Subtitle) (16) Sequential Tests Under Weibull Distribution.		5. TYPE OF REPORT & PERIOD COVERED (9) Technical Report,
		6. PERFORMING ORG. REPORT NUMBER Q-7
7. AUTHOR(s) (10) P. M. Ghare		8. CONTRACT OR GRANT NUMBER(s) DAAG-29-78-G-0172
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Polytechnic Institute & State University Blacksburg, Virginia 24061		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE (11) December 1980
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) A1 9 (19) 15147-M (12) 22		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acceptance procedures, sequential probability ratio test, military standards, Weibull distribution.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The objective of this paper is to develop acceptance procedures based on a sequential probability ratio test when the system failures follow Weibull probability distribution. The acceptance procedures in both Military Handbook 108 and Mil-Std-781C are based on esponential distribution of failures. (OVER)

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S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

401206

ABSTRACT (Con't)

Exponential distribution implies a constant hazard rate. The Weibull distribution is much more comprehensive and, with suitable choice of parameters, can describe constant, increasing or decreasing hazard rates.

The specifications are usually written in terms of the scale parameter α, the shape parameter θ being determined by the nature of the system under test. When N units are being tested and n units have failed at time t, the acceptance is based on the monitoring of a statistic σ_t given as

$$\sigma_t = n(\log q_1 - \log q_0) + \frac{\alpha_1 - \alpha_0}{\theta+1} \left| \sum_{i=1}^{n} t_i^{\theta+1} + (N-n)t^{\theta+1} \right|$$

where q_0 and q_1 are the desirable and unacepptable values of the scale parameters. The indicated decision is to

accept if $\sigma_t \geq$ a constant, $\log Z_1$

reject if $\sigma_t \leq$ another constant, $\log Z_2$

and to continue testing if σ_t remains within the two limits.

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